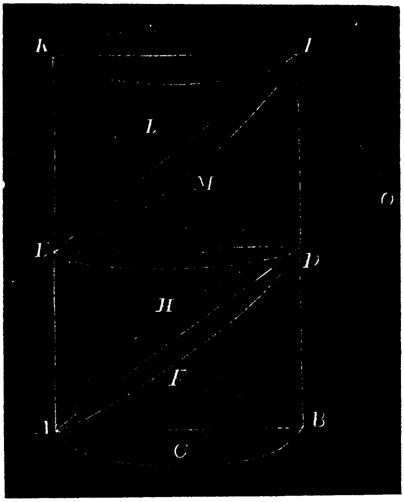
DETERMINATION OF THE AREA OF AN ELLIPSE GEOMETRICALLY, THAT OF THE CIRCLE BEING KNOWN.

BY OCTAVIAN L. MATHIOT, BALTIMORE, MD.

LET $ABDE$ represent any upright circular cylinder whose diameter is AB and perpendicular height BD. Let the cylinder be cut by a plane through D and A, the cutting edge of which is parallel to the plane of the base; then will the section $AFDH$ be an ellipse whose transverse axis is AD and whose conjugate axis $FH = AB$, the diameter of the cylinder; and it is evident from the figure that this ellipse divides the cylinder into two equal parts.



Prolong the cylinder to IK so that $DI = DB$, and through IE pass a plane parallel to the plane through DA, then will the section $EMIL$ be an ellipse similar and equal to the ellipse $AFDH$

Let the diameter of the base, $AB = b$ and the height $BD = h$, then is $AD = \sqrt{(b^2 + h^2)} = a$, say; hence a and b are he axes of the ellipse.

Produce AD to meet a perpendicular from I in O, then will the triangles ABD and IOD be similar, and we shall have $AD : AB :: ID : IO$, or, $a : b :: h : p$, where $p = IO$ the perpendicular distance between the sections through AD and EI; hence

$$p = \frac{bh}{a}; \therefore \mathcal{A}\frac{bh}{a} = \tfrac{1}{4}\pi b^2 h, \tag{1}$$

where $\mathcal{A}$ = the area of the ellipse;

$$\therefore \mathcal{A} = \tfrac{1}{4}\pi ab. \tag{2}$$

The equality expressed by (1) is obvious from the construction of the Fig., and the result, in (2) is the recognized expression for the area of an ellipse, but, so far as I know, it has hitherto been obtained from a higher analysis than the foregoing.

In giving the above formula for the mensuration of the ellipse, a well-known author says; "this rule and the one for the following problem (the parabola) cannot be demonstrated without the aid of principles not yet considered" &c. (See Davies' Legendre, for the use of schools, p. 284.)